

Warm-up:

Complete the Square # 1-4

1) $x^2 + 18x + \underline{81}$ $\frac{18}{2}^2 = 9^2 = 81$

2) $x^2 + 4x + 2 = 0$
 $(x+2)^2 = 2$

3) $-2x^2 + 12x = -20$
 $\frac{-2x^2 + 12x}{2} = \frac{-20}{2}$ $x^2 + 6x + 9 = -20 + 9$ $(x+3)^2 = -11$

4) $x^2 - 20x + 30$
 $(x-10)^2 = 70$

$$\frac{5x-1}{x+8} - \frac{3x+4}{x+8} = \frac{(5x-1)-(3x+4)}{x+8} = \frac{5x-1-3x-4}{x+8} = \frac{5x-3x-1-4}{x+8} = \frac{2x-5}{x+8}$$

$$\frac{x+4}{2x} - \frac{x-1}{x^2} = \left(\frac{x+4}{2x}\right)\left(\frac{x}{x}\right) - \left(\frac{x-1}{x^2}\right)\left(\frac{2}{2}\right)$$

$$= \frac{x^2+4x}{2x^2} - \frac{2x-2}{2x^2}$$


$$= \frac{(x^2+4x)-(2x-2)}{2x^2}$$

$$= \frac{x^2+4x-2x+2}{2x^2}$$

$$= \frac{x^2+2x+2}{2x^2}$$


Your Turn

$$\frac{3x}{x^2+3x-10} - \frac{6}{x^2+3x-10}$$



$$= \frac{3}{x+5}$$

$$\frac{4x}{2x-1} - \frac{5}{x-6}$$

$$\frac{4x}{2x-1} - \frac{5}{x-6} = \left(\frac{4x}{2x-1}\right)\left(\frac{x-6}{x-6}\right) - \left(\frac{5}{x-6}\right)\left(\frac{2x-1}{2x-1}\right)$$


$$= \frac{4x^2-34x+5}{(2x-1)(x-6)}$$


Multiply: $\frac{x^2-5x-24}{x^2-7x-18} \cdot \frac{x^2+13x+22}{x^2-64}$

$$= \frac{(x+3)(x-8)}{(x+2)(x-9)} \cdot \frac{(x+2)(x+11)}{(x+8)(x-8)}$$

$$= \frac{(x+3)(x-8)}{(x+2)(x-9)} \cdot \frac{(x+2)(x+11)}{(x+8)(x-8)}$$

$$= \frac{(x+3)(x+11)}{(x-9)(x+8)}$$

Your Turn

$$\frac{x^2-8x+12}{x^2-16} \cdot \frac{4x+16}{x^2-4x+4}$$


$$= \frac{4(x-6)}{(x-4)(x-2)}$$


$$\frac{x^2-6x-16}{x^2+4x-21} \div \frac{x^2+9x+14}{x^2-8x+15} = \frac{x^2-6x-16}{x^2+4x-21} \cdot \frac{x^2-8x+15}{x^2+9x+14}$$

$$= \frac{(x+2)(x-8)}{(x-3)(x+7)} \cdot \frac{(x-3)(x-5)}{(x+2)(x+7)}$$

$$= \frac{(x+2)(x-8)(x-3)(x-5)}{(x-3)(x+7)(x+2)(x+7)}$$

$$= \frac{(x-8)(x-5)}{(x+7)^2}$$

Divide: $\frac{x^2+6x-7}{x^2+x-30} \div \frac{x^2-49}{x+6}$



$$= \frac{x-1}{(x-5)(x-7)}$$

Unit 2 ~ Quadratics

Objective: A.SSE.3b

Day 4: Applications of Completing the Square - (Vertex Form)

Application #1:

Converting Quadratics from Standard Form to Vertex Form

Remember:

*Standard Form: $y = ax^2 + bx + c$

*Vertex Form: $y = a(x - h)^2 + k$

Vertex: (h, k) Focus: $(h, k+c)$ Directrix: $(y = k - c)$ $a = \frac{1}{4c}$

Example 1:

What is the vertex form of $y = x^2 - 4x - 9$?

$$x^2 - 4x = 9$$

$$x^2 - 4x + \left(\frac{-4}{2}\right)^2 = 9 + \left(\frac{-4}{2}\right)^2$$

$$x^2 - 4x + 4 = 13$$

$$(x-2)^2 = 13$$

Vertex form: $y = a(x-h)^2 + k$

$$y = (x-2)^2 - 13$$

$h=2$ $k=-13$

(h, k)
vertex: $(2, -13)$

$h, k+c$

focus: $(2, -13+25)$
 $(2, 12.75)$

Steps

- Complete the square.
- Write in vertex form
- Find the value of 'c'
- $a = \frac{1}{4c}$
- Find the focus and directrix

$$a=1$$

$$4c \cdot 1 = \frac{1}{4c} \cdot 4c$$

$$\frac{4c}{4} = \frac{1}{4}$$

$$c = \frac{1}{4} \text{ or } .25$$

$$y = k - c$$

$$y = -13 - .25$$

$$y = -13.25$$

Example 2:

Rewrite the equation $3x^2 - 12x = -9$ in vertex form.

$$3(x^2 - 4x) = -3$$

$$3(x^2 - 4x + \left(\frac{-4}{2}\right)^2) = -3 + \left(\frac{-4}{2}\right)^2$$

$$3(x^2 - 4x + 4) = 1 = 3(x-2)^2 - 1$$

Vertex form: $y = 3(x-2)^2 - 1$

$h=2$ $k=-1$ $c=.08$

$4c \cdot 3 = \frac{1}{4c} \cdot 4c$

$\frac{12c}{12} = \frac{1}{12}$ $c = \frac{1}{12} \cdot .08$

(h, k)
vertex: $(2, -1)$ focus: $(2, -.92)$ directrix: $y = -1.08$

$y = k - c$

Write the equation in Vertex Form

$$x^2 + 10x - 5$$

$$x^2 + 10x - 5$$

$$y = x^2 + 10x - 5 + 5$$

$$x^2 + 10x = 5$$

$$x^2 + 10x + \left(\frac{10}{2}\right)^2 = 5 + \left(\frac{10}{2}\right)^2$$

$$x^2 + 10x + 25 = 5 + 25$$

$$x^2 + 10x + 25 = 30$$

$$y = (x+5)^2 - 30$$

$$h = -5$$
 $k = -30$

Vertex form: $(-5, -30)$

Focus: $(-5, -30+25)$
 $(-5, -29.75)$

Directrix: $y = 30 - 25$
 $y = 5$

Your Turn:

Write the equation $\frac{2x^2}{2} + \frac{8x}{2} = \frac{-10}{2}$

$$2(x^2 + 4x) = -5$$

$$2\left(x^2 + 4x + \left(\frac{4}{2}\right)^2\right) = -5 + \left(\frac{4}{2}\right)^2$$

$$2(x^2 + 4x + 4) = -5 + 4$$

$$2(x+2)^2 = -1$$

$$2(x+2)^2 = -1$$

$$h = -2 \quad k = -1$$

$$a = \frac{1}{4c}$$

$$8c = 1$$

$$c = \frac{1}{8} = 0.125$$

Vertex $(-2, -1)$, focus $(-2, -1.875)$, directrix: $y = -1.125$

